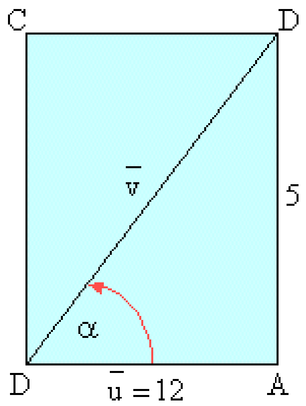
1 . Diketahui persegi panjang OABC dengan panjang OA = 12 dan AB = 5. Jika

$OA = \vec{u}$ dan $OB = \vec{v}$ maka $\vec{u} \cdot \vec{v}$

A . 13
B . 60
C . 144
D . 149
E . 156

Kunci : C

Penyelesaian :



$$\bar{v} = \sqrt{12^2 + 5^2}$$
$$= \sqrt{144 + 25}$$
$$= 13$$
$$\bar{u} \cdot \bar{v} = |\bar{u}| \cdot |\bar{v}| \cos\alpha$$
$$= 12 \cdot 13 \cdot \frac{12}{13}$$
$$= 144$$

2 . Jika, Jika $\alpha + \beta = \frac{\pi}{6}$ dan $\cos\alpha\cos\beta = \frac{3}{4}$, maka $\cos(\alpha - \beta) = \ldots\ldots\ldots$ maka cos ($\propto$ - β) = ...

A . $\frac{1}{9} - \frac{\sqrt{3}}{2}$
B . $\frac{3}{2} + \frac{\sqrt{3}}{2}$
C . $\frac{3}{4} - \frac{\sqrt{3}}{2}$
D . $\frac{3}{2} - \frac{\sqrt{3}}{2}$
E . $\frac{\sqrt{3}}{2}$

Kunci : D

Penyelesaian :

$$\alpha + \beta = \frac{\pi}{6} \Rightarrow \cos(\alpha + \beta) = \cos\frac{\pi}{6} = \frac{1}{2}\sqrt{3}$$
$$\cos\alpha\cos\beta = \frac{3}{4} \Rightarrow 2\cos\cos\alpha\cos\beta$$
$$\Rightarrow 2\cos\alpha\cos\beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$$
$$\Rightarrow 2\left(\frac{3}{4}\right) = \frac{1}{2}\sqrt{3} + \cos(\alpha - \beta)$$
$$\Rightarrow \cos(\alpha - \beta) = \frac{3}{2} - \frac{1}{2}\sqrt{3}$$

3 . Diketahui kubus ABCD.EFGH dengan rusuk 4. Titik T pada perpanjangan CG, sehingga CG = GT. Jika sudut antara TC dan bidang BDT adalah, maka tg $\propto$ = ...

A . $\sqrt{2}$

B . $\dfrac{\sqrt{2}}{2}$

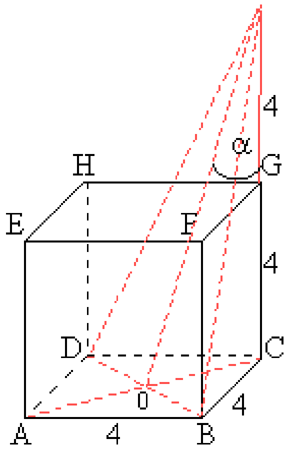
C . $\dfrac{\sqrt{2}}{3}$

D . $\dfrac{\sqrt{2}}{4}$

E . $\dfrac{\sqrt{2}}{6}$

Kunci : D

Penyelesaian :



$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{4^2 + 4^2}$$
$$= 4\sqrt{2}$$
$$OC = \tfrac{1}{2} AC = 2\sqrt{2}$$
$$\tan \alpha = \frac{OC}{CT} = \frac{2\sqrt{2}}{8} = \frac{\sqrt{2}}{4}$$

4 . Jika $a = \lim\limits_{y \to \infty} \{(2y+1)) - \sqrt{4y^2 - 4y + 3}\}$

maka untuk $0 < x < \dfrac{\pi}{2}$, deret

1 + ${}^{a}\log \sin x$ + ${}^{a}\log^{2} \sin x$ + ${}^{a}\log^{3} \sin x$ + ... konvergen hanya pada selang ....

A . $\dfrac{\pi}{6} < x < \dfrac{\pi}{2}$

B . $\dfrac{\pi}{6} < x < \dfrac{\pi}{4}$

C . $\dfrac{\pi}{4} < x < \dfrac{\pi}{3}$

D . $\dfrac{\pi}{4} < x < \dfrac{\pi}{2}$

E . $\dfrac{\pi}{3} < x < \dfrac{\pi}{2}$

Kunci : A

Penyelesaian :

$$a = \lim_{y\to\infty} \{(2y+1) - \sqrt{4y^2 - 4y + 3}\}$$

$$\Rightarrow a = \lim_{y\to\infty} \{(2y+1) - \sqrt{4y^2 - 4y + 3}\} . \frac{(2y+1) + \sqrt{4y^2 - 4y + 3}}{(2y+1) + \sqrt{4y^2 - 4y + 3}}$$

$$= \lim_{y\to\infty} \frac{(2y+1)^2 - (4y^2 - 4y + 3)}{(2y+1) + \sqrt{4y^2 - 4y + 3}}$$

$$= \lim_{y\to\infty} \frac{4y^2 + 4y - 1 - 4y^2 - 4y + 3}{(2y+1) + \sqrt{4y^2 - 4y + 3}}$$

$$= \lim_{y\to\infty} \frac{8y - 2}{(2y+1) + \sqrt{4y^2 - 4y + 3}}$$

$$= \lim_{y\to\infty} \frac{\frac{8y}{y} - \frac{2}{y}}{\frac{2y}{y} + \frac{1}{y} + \sqrt{\frac{4y^2}{y^2} - \frac{4y}{y^2} + \frac{3}{y^2}}}$$

$$= \lim_{y\to\infty} \frac{8 - \frac{2}{y}}{2 + \frac{1}{y} + \sqrt{4 - \frac{4}{y} + \frac{3}{y^2}}}$$

$$= \frac{8}{2 + 2\sqrt{2}} = 2$$

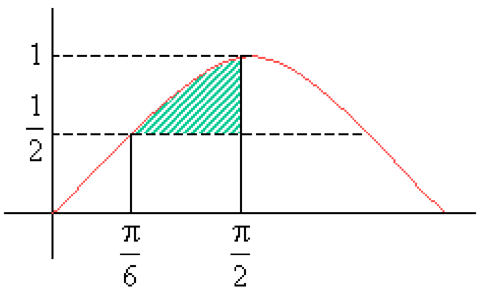
$a = 2$

$1 + {}^{a}\log \sin x + {}^{a}\log^{2} \sin x + {}^{a}\log^{3} \sin x + ...$

konvergen jika $-1 < r < 1$

$\Rightarrow -1 < {}^{2}\log \sin x < 1$

$\Rightarrow \frac{1}{2} < \log \sin x < 2$



Jadi $\frac{\pi}{6} < x < \frac{\pi}{2}$

5 . Bila jarak sesuatu titik dari suatu posisi P pada setiap waktu t diberikan sebagai s(t) = A sin 2t, A>0, maka kecepatan terbesar diperoleh pada waktu t = ...

A . $\frac{k}{2}\pi$, $k = 0, 1, 2, 3, 4 \ldots$

B . $\frac{k}{2}\pi$, $k = 1, 3, 5 \ldots$

C . $\frac{k}{2}\pi$, $k = 0, 2, 4, 6 \ldots$

D . $\frac{k}{2}\pi$, $k = \frac{1}{2}, \frac{5}{2}, \frac{9}{2} \ldots$

E . $\frac{k}{2}\pi$, $k = \frac{3}{2}, \frac{7}{2}, \frac{11}{2} \ldots$

Kunci : A
Penyelesaian :
S(t) = A sin 2t : A>0
V(t) = S'(t) = 2 A cos 2t
Kecepatan terbesar didapat jioka | v'(t) | = 0
$\Rightarrow$ | -4 A sin 2t | = 0
$\Rightarrow$ | sin 2t | = 0
$\Rightarrow$ sin 2t = sin 0
$\Rightarrow$ 2t = K. 2π
$\Rightarrow$ t = K π atau
$\Rightarrow$ 2t = π + K . 2π

$$t = \frac{\pi}{2} + K\pi$$

$$K = 0 \rightarrow t = 0 \text{ atau } \frac{\pi}{2}$$

$$K = 1 \rightarrow t = 2\pi \text{ atau } \frac{3}{2}\pi$$

$$K = 2 \rightarrow t = 2\pi \text{ atau } \frac{5}{2}\pi \text{ dan seterusnya}$$

$$\text{jadi } t = \frac{k}{2}\pi, K = 1, 2, 3........$$

6 . Garis g melalui titik (2,4) dan menyinggung parabola y^2 = 8x. Jika garis h melalui (0,0) dan tegak lurus pads garis g, maka persamaan garis h adalah ......

A . x + y = 0
B . x - y = 0
C . x + 2y = 0
D . x - 2y = 0
E . 2x + y = 0

Kunci : A
Penyelesaian :
g : y^2 = 8x
$\Rightarrow$ 2y dy = 8x

$$g : y^2 = 8x$$

$$\Rightarrow 2y\,dy = 8\,dx$$

$$\Rightarrow \frac{dy}{dx} = m_g = \frac{8}{2y} = \frac{4}{y} = \frac{4}{4} = 1$$

$$h \perp g \Rightarrow m_g . m_h = -1$$

$$\Rightarrow 1.m_h = -1$$

$$\Rightarrow \quad m_h = -1$$

$$\text{h melalui } (0,0) \Rightarrow y = m_h . x$$

$$\Rightarrow y = -x$$

$$\Rightarrow y + x = 0$$

7 . Akar-akar persamaan kuadrat (p-2)x² + 4x + (p + 2) = 0 adalah α dan β.
Jika $\alpha \beta^2 + \beta \alpha^2$ = -20, maka p =

A . -3 atau $\frac{-6}{5}$

B . -3 atau $\frac{-5}{6}$

C . -3 atau $\frac{5}{6}$

D . -3 atau $\frac{-6}{5}$

E . 3 atau $\frac{6}{5}$

Kunci : E

Penyelesaian :

$(p-2)x^2 + 4x + (p + 2) = 0$

akarnya $\propto$ dan β

$(p-2)x^2 + 4x + (p+2) = 0$

akarnya α dan β

$\Rightarrow \alpha + \beta = \frac{-4}{p-2}$

$\Rightarrow \alpha . \beta = \frac{p+2}{p-2}$

$\Rightarrow \alpha\beta^2 + \beta\alpha^2 = -20$

$\Rightarrow \alpha\beta(\alpha+\beta) = -20$

$\Rightarrow (\frac{p+2}{p-2})(\frac{-4}{p-2}) = -20$

$\Rightarrow \frac{-4(p+2)}{(p-2)^2} = -20$

$\Rightarrow -4(4+2) = -20(p-2)^2$

$\Rightarrow p + 2 = 5(p^2 - 4p + 4)$

$\Rightarrow p + 2 = 5p^2 - 20p + 20$

$\Rightarrow 5p^2 - 21p + 18 = 0$

$\Rightarrow (5p-6)(p-3) = 0$

$p = \frac{6}{5}$ atau $p = 3$

8 . Diketahui $\frac{dF}{dx} = ax + b, F(0) - F(-1) = 3, F(1) - F(0) = 5$, maka $a + b = \ldots\ldots$

A . 8

B . 6

C . 2

D . -2

E . -4

Kunci : B

Penyelesaian :

$$\frac{dF}{dx} = ax + b \Rightarrow F(x) = \int (ax+b)\,dx$$

$$= \frac{a}{2}x^2 + bx + c$$

$$F(0) - F(-1) = \frac{a}{2}(0)^2 + b(0) + c(\frac{a}{2}(-1)^2) + b(-1) + c$$

$$\Rightarrow 3 = c - \frac{a}{2} + b - c$$

$$\Rightarrow 3 = b - \frac{a}{2} \text{................ (1)}$$

$$F(1) - F(0) = \frac{a}{2}(1)^2 + b(1) + c - c$$

$$5 = \frac{a}{2} + b \text{........(2)}$$

$$b = 4 \rightarrow 3 = 4 - \frac{a}{2}$$

$$\rightarrow a = 2$$

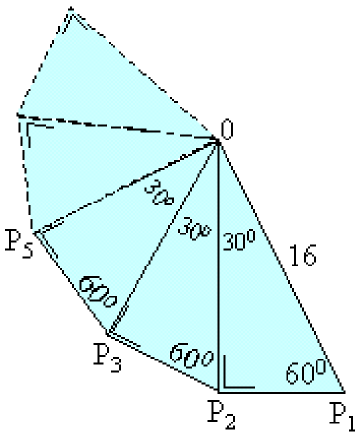
$$\text{Jadi } a + b = 2 + 4 = 6$$

9 . Diketahui sebuah segitiga OP_1P_2 dengan sudut siku-siku pada P_2 dan sudut puncak 30° pada O. Dengan OP_2 sebagai sisi miring dibuat pula segitiga siku-siku OP_2P_3 dengan sudut puncak P_2OP_3 sebesar 30°. Selanjutnya dibuat pula segitiga siku-siku OP_3P_4 dengan OP_3 sebagai sisi miring dan sudut puncak P_3OP_4 sebesar 30°. Proses ini dilanjutkan terus-menerus. Jika OP_1 = 16, maka jumlah luas seluruh segitiga adalah ..... maka jumlah luas seluruh segitiga adalah ......

A . $64\sqrt{3}$
B . 128
C . $128\sqrt{3}$
D . 256
E . $256\sqrt{3}$

Kunci : C

Penyelesaian :



ΔOP_1P_2

$\sin 30^0 = \dfrac{P_1P_2}{16} = \dfrac{1}{2} \Rightarrow P_1P_2 = 8$

$\sin 60^0 = \dfrac{OP_2}{16} = \dfrac{\sqrt{3}}{2} \Rightarrow OP_2 = 8\sqrt{3}$

$L\Delta OP_1P_2 = \dfrac{1}{2}.P_1P_2.OP_1.\sin 60^0$

$= \dfrac{1}{2}.8.16.\dfrac{1}{2}\sqrt{3} = 32\sqrt{3}$

ΔOP_2P_3

$\sin 30^0 = \dfrac{P_1P_2}{8\sqrt{3}} = \dfrac{1}{2} \Rightarrow P_2P_3 = 4\sqrt{3}$

$L\Delta OP_2P_3 = \dfrac{1}{2}.P_2P_3.OP_2.\sin 60^0$

$= \dfrac{1}{2}.4\sqrt{3}.8\sqrt{3}.\dfrac{1}{2}\sqrt{3} = 24\sqrt{3}$

$32\sqrt{3}, 24\sqrt{3}$,........

$$S = \frac{32\sqrt{3}}{1 - \frac{3}{4}} = 128\sqrt{3}$$

10 . Himpunan jawaban pertidaksamaan $^3\log x + {^3\log} (2x - 3) < 3$ adalah ....

A . $\{x | x > \frac{3}{2}\}$

B . $\{x | x > \frac{9}{2}\}$

C . $\{x | 0 < x > \frac{9}{2}\}$

D . $\{x | \frac{3}{2} < x < \frac{9}{2}\}$

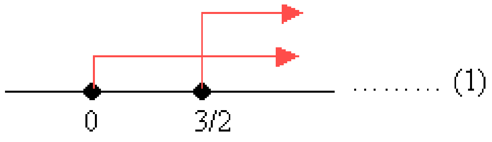
E . $\{x | -3 < x < \frac{9}{2}\}$

Kunci : D

Penyelesaian :

$^3\log x + {^3\log} (2x - 3) < 3$

syarat $x>0$, dan $2x - 3 > 0$



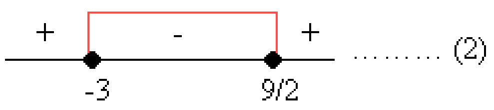
$^3\log x + {^3\log} (2x - 3) < {^3\log} 27$

$\Rightarrow {^3\log} (2x^2 - 3) < {^3\log} 27$

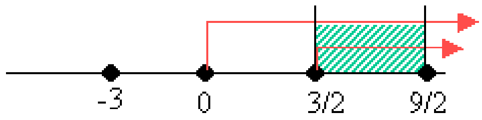
$\Rightarrow 2x^2 - 3x < 27$

$\Rightarrow 2x^2 - 3x - 27 < 0$

$\Rightarrow (2x - 9)(x + 3) < 0$



(1) dan (2) disatukan



jadi $\{x \mid \frac{3}{2} < x < \frac{9}{2}\}$